Hypothesis Testing for Means

Lecture 33
Sections 10.1-10.2

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Outline

Introduction

- In Chapter 10 we will ask the same basic questions as in Chapter 9, except they will concern the mean.
  - Find an estimate for the mean.
  - Test a hypothesis about the mean.
- We will begin with hypothesis testing.

The Steps of Testing a Hypothesis

1. State the null and alternative hypotheses.
2. State the significance level.
3. Give the test statistic, including the formula.
4. Compute the value of the test statistic.
5. Compute the p-value.
6. State the decision.
7. State the conclusion.

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The Hypotheses

- The null and alternative hypotheses will be statements concerning μ.
- Null hypothesis.
$$H_0 : \mu = \mu_0.$$
- Alternative hypothesis (choose one).
  - $H_1 : \mu < \mu_0$.
  - $H_1 : \mu > \mu_0$.
  - $H_1 : \mu \neq \mu_0$.

The Hypotheses

- See Example 10.1, p. 616.
- The hypotheses are

$$H_0 : \mu = 15 \text{ mg}$$
$$H_1 : \mu < 15 \text{ mg}$$

Level of Significance

- The level of significance is the same as before.
- If the value is not given, assume that $\alpha = 0.05$.

The Test Statistic

- The choice of test statistic will depend on the sample size and what is known about the population. (Details to follow.)
- If we assume that σ is known and that either
  - The sample size n is at least 30, or
  - The population is normal,

  then the Central Limit Theorem for Means will apply.

The Sampling Distribution of $\overline{x}$

- If the population is normal, then the distribution of $\overline{x}$ is also normal, with mean μ_0 and standard deviation $\sigma/\sqrt{n}$

$$\overline{x} \text{ is exactly } N\left(\mu_0, \frac{\sigma}{\sqrt{n}}\right)$$

  for any sample size (no matter how small).
- This assumes that σ is known.

The Sampling Distribution of $\overline{x}$

- Therefore, the test statistic is

$$Z = \frac{\overline{x} - \mu_0}{\sigma/\sqrt{n}}.$$

- It is exactly standard normal.

The Sampling Distribution of $\overline{x}$

- On the other hand, if
  - The population is not normal,
  - But the sample size n is at least 30,

  then the distribution of $\overline{x}$ is approximately normal, with mean μ_0 and standard deviation $\frac{\sigma}{\sqrt{n}}$.

$$\overline{x} \text{ is approximately } N\left(\mu_0, \frac{\sigma}{\sqrt{n}}\right).$$

- We are still assuming that σ is known.

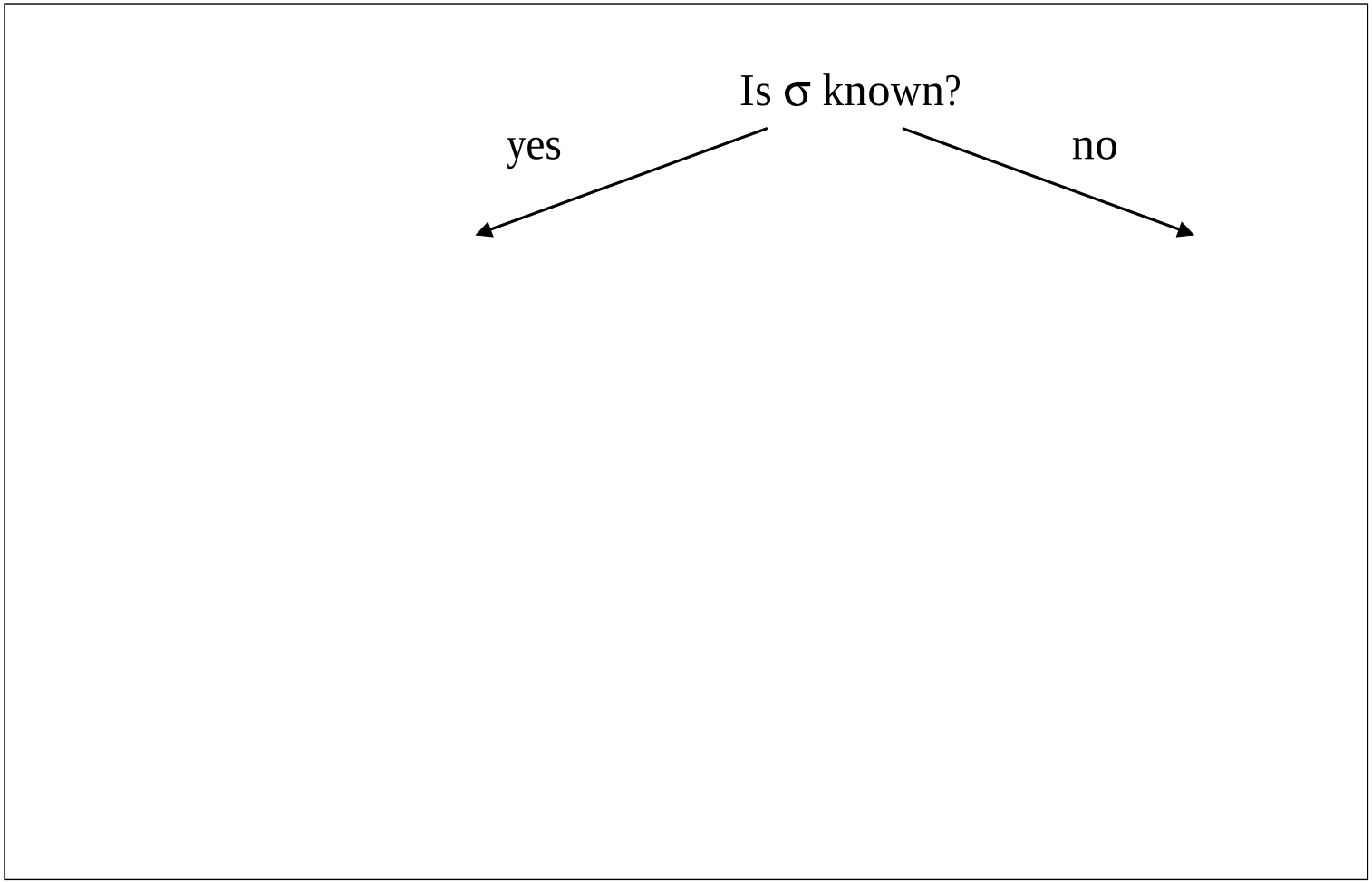
The Sampling Distribution of $\overline{x}$

- Therefore, the test statistic is

$$Z \approx \frac{\overline{x} - \mu_0}{\sigma/\sqrt{n}}.$$

- It is approximately standard normal.
- The approximation is good enough that we can use `normalcdf`.

The Decision Tree



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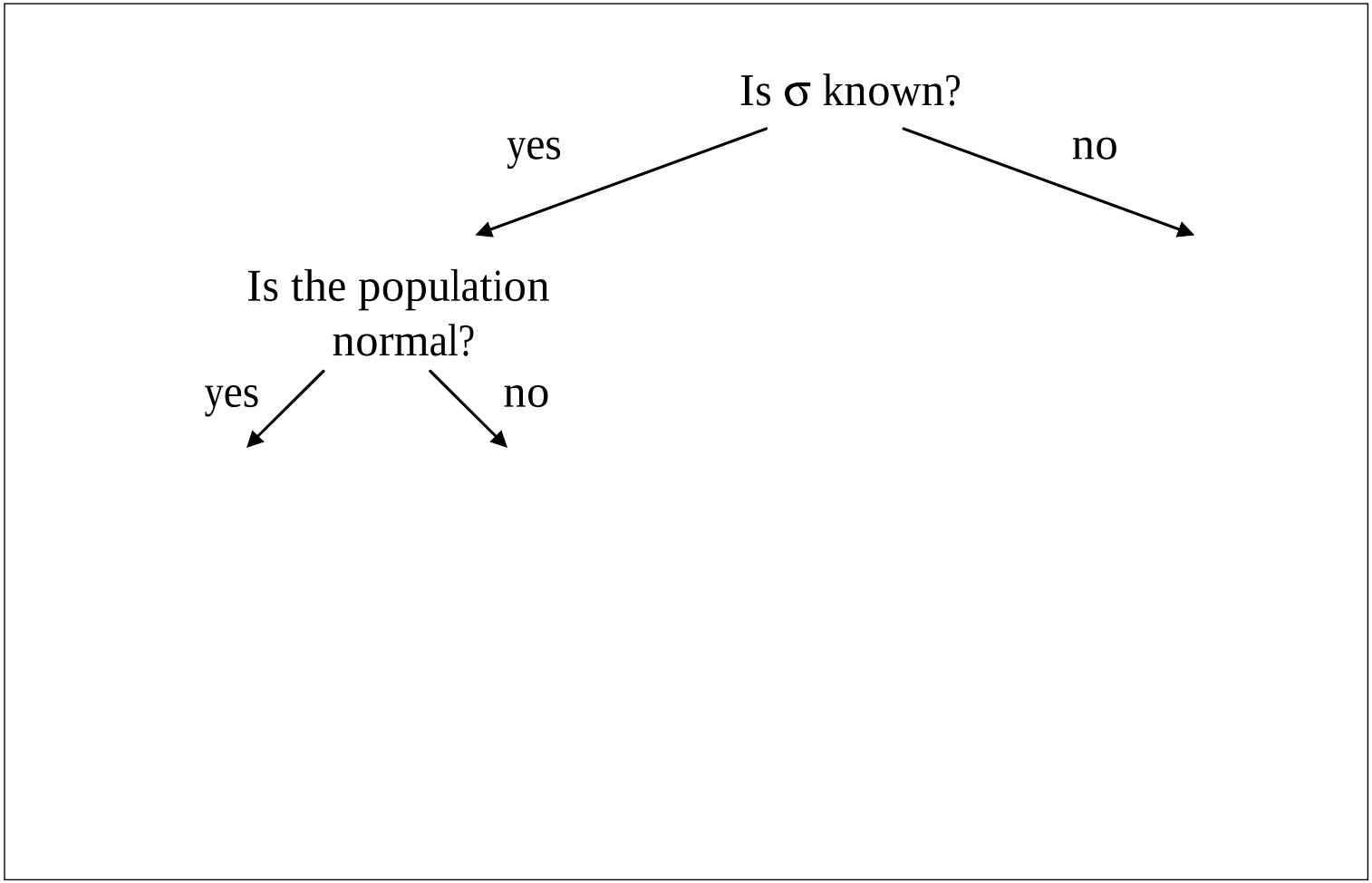

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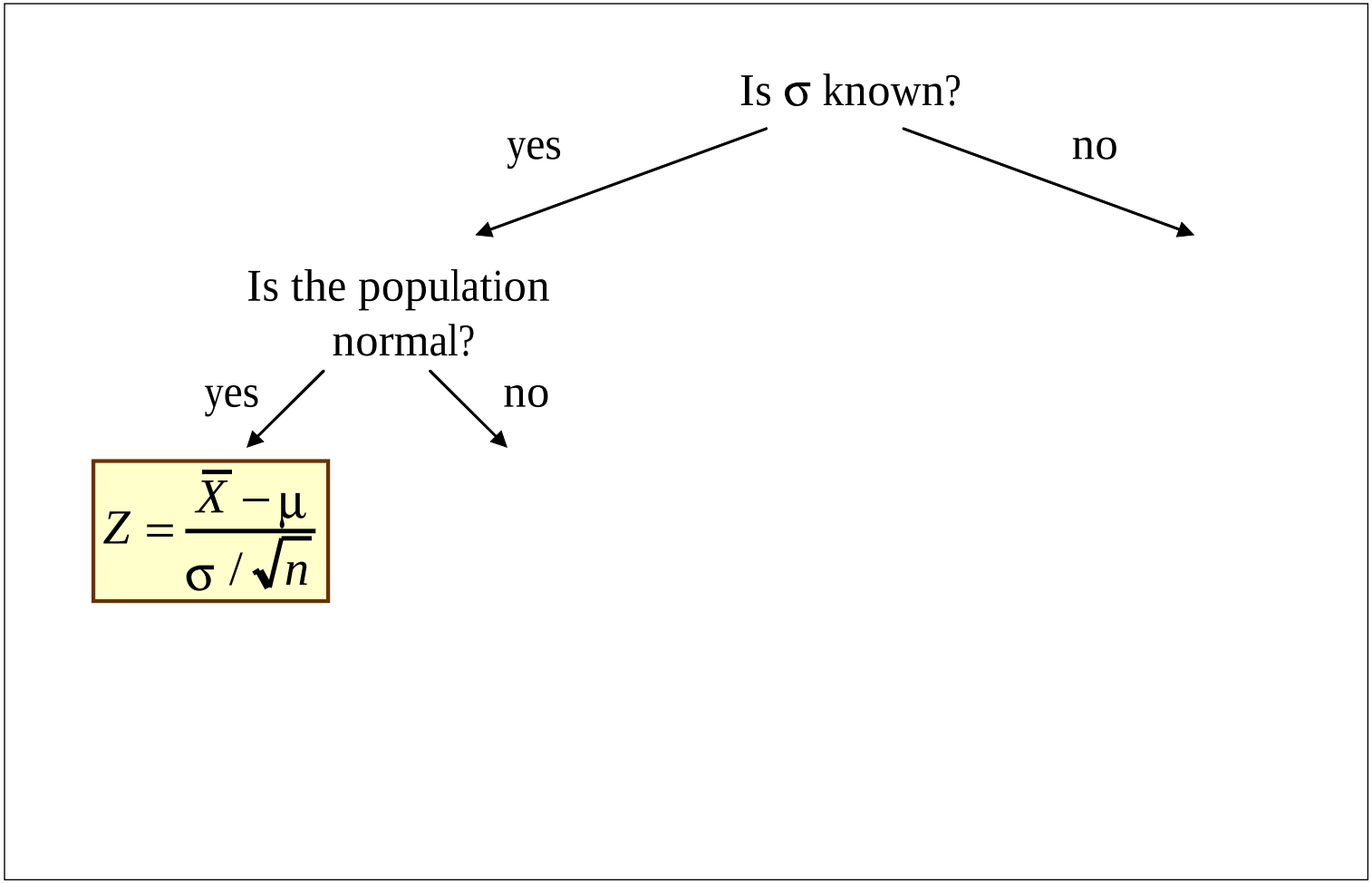

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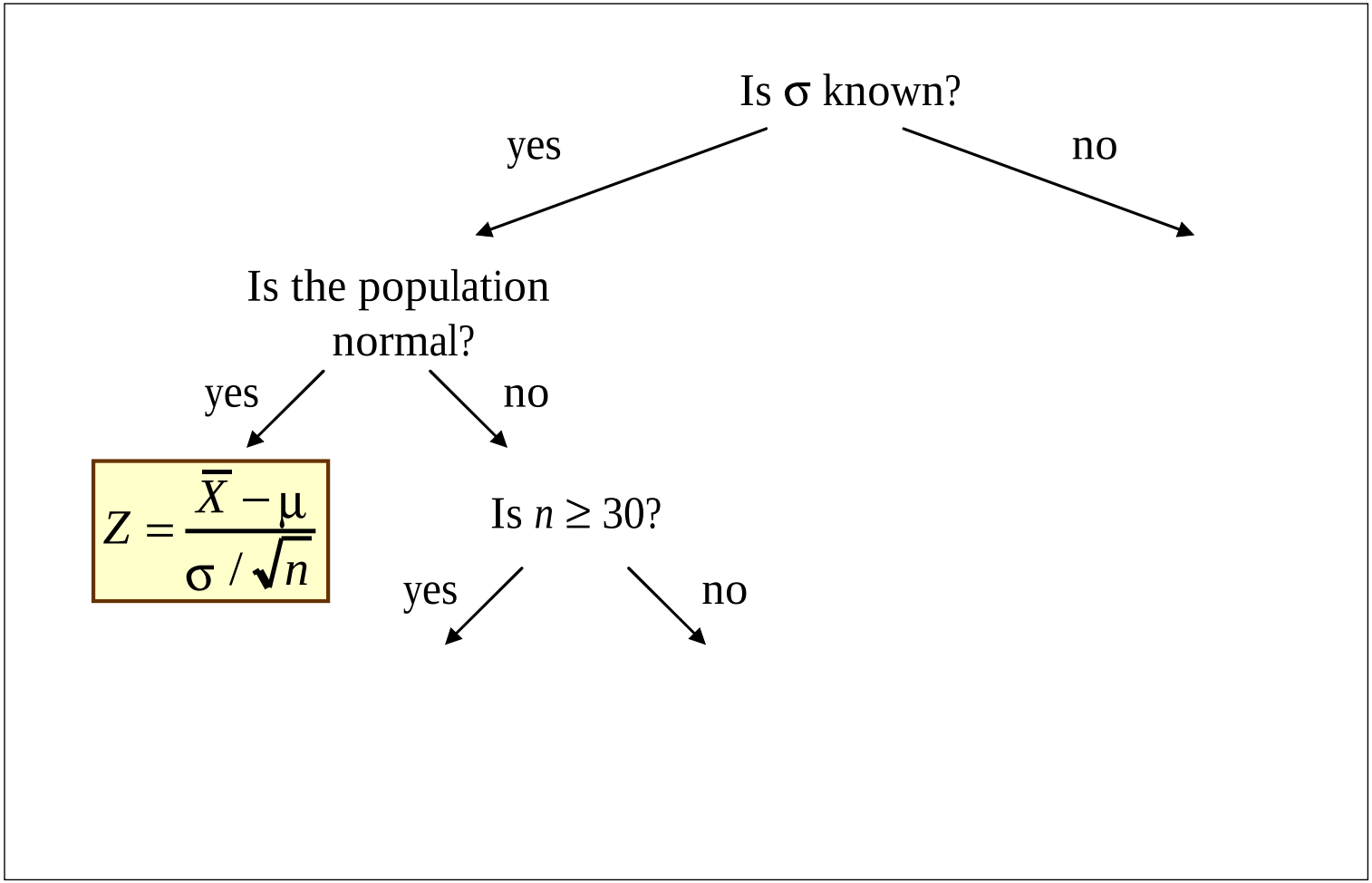

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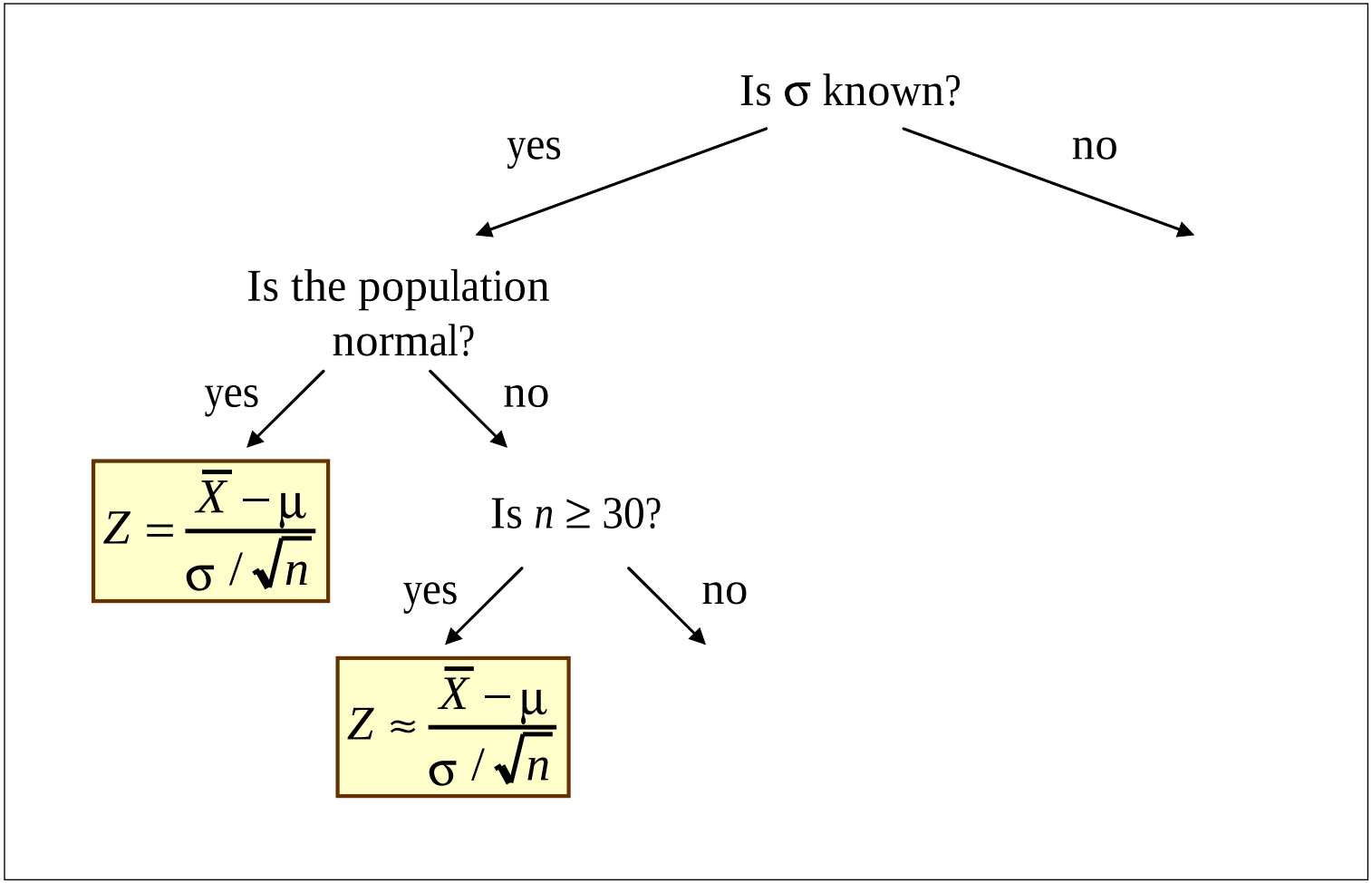

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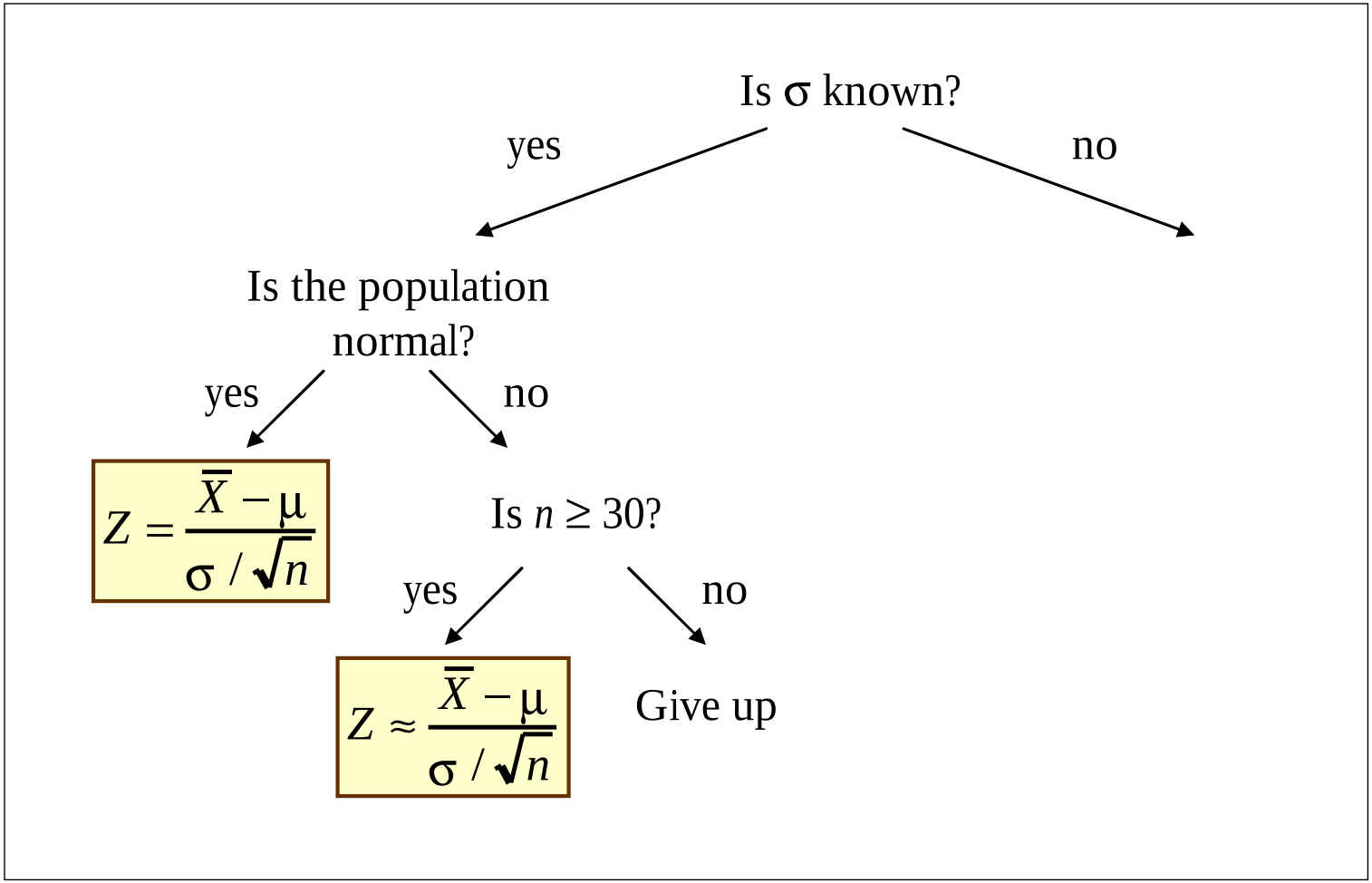

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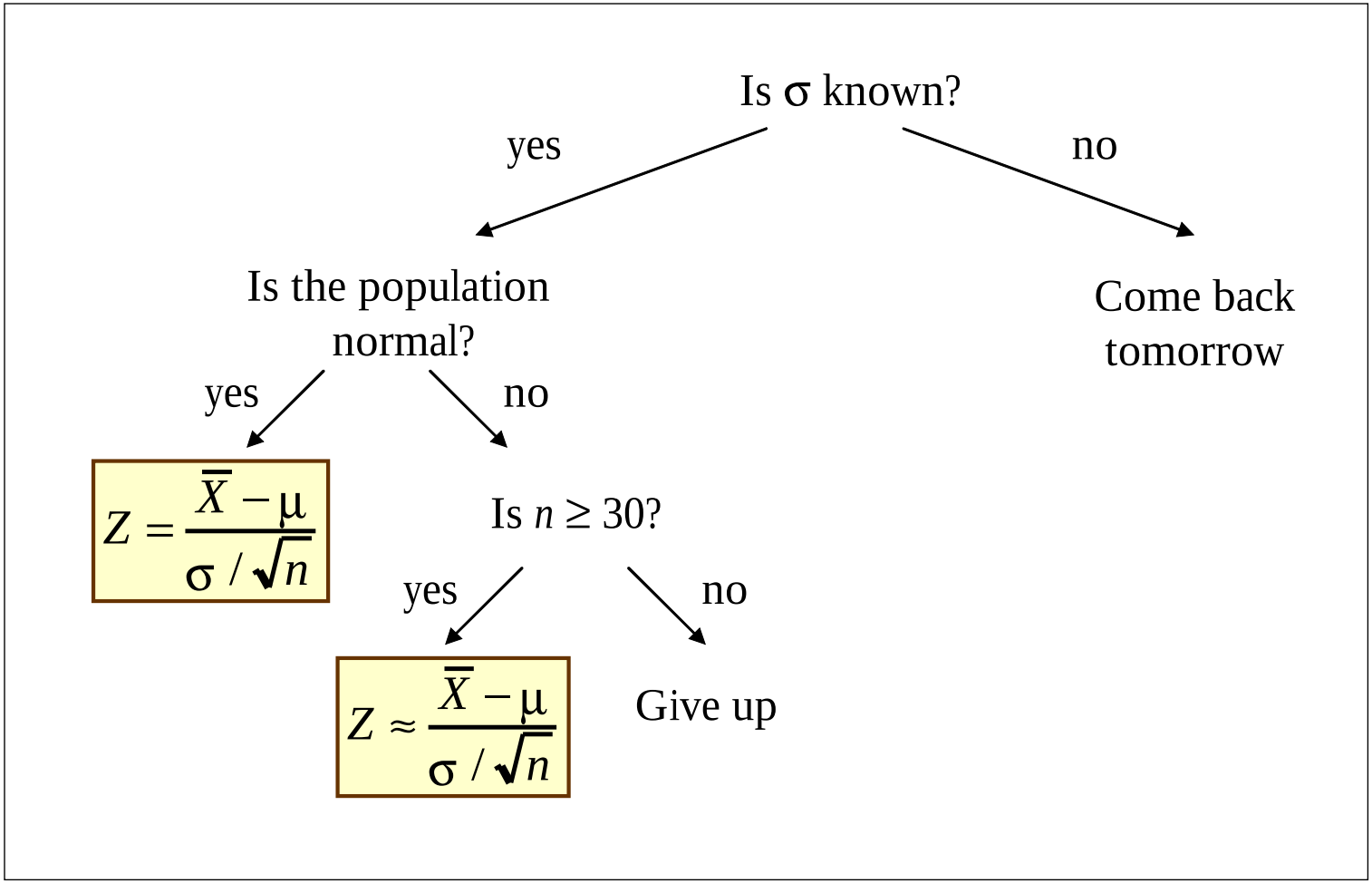

Calculate the Value of the Test Statistic

- In our sample, we find that $\overline{x} = 12.528$.
- We are assuming that $\sigma = 4.8$.
- Therefore,

$$Z = \frac{12.528 - 15}{4.8/\sqrt{25}} = \frac{-2.472}{0.96} = -2.575.$$

Compute the p-Value

- The p-value is $P(\overline{x} < -2.575)$.
- Use `normalcdf(-E99,-2.575)` $= 0.005012$.
- Therefore, p-value $= 0.005012$.

Decision

- Because the *p*-value is less than α, we will reject the null hypothesis.

Conclusion

- We conclude that the carbon monoxide content of cigarettes is lower today than it was in the past.

Exercise

- Work the example “Completing a Maze" on page 620:
  - Under normal circumstances, mice complete a maze in an average time of 18 seconds, with a standard deviation of 2 seconds.
  - A researcher introduces loud noises. Will this cause the mice to run the maze faster?
  - A sample of 10 mice has an average time of 17 seconds.
  - Assume that their run times are normally distributed.
  - Test the hypothesis at the 10% level that the average run time is less when there is loud noise.

Hypothesis Testing on the TI-83

- Press `STAT`.
- Select `TESTS`.
- Select `Z-Test`. Press `ENTER`. A window appears requesting information.
- Select `Data` if you have the sample data entered into a list.
- Otherwise, select `Stats`.

Hypothesis Testing on the TI-83

- Assuming you selected `Stats`,
- Enter μ_0, the hypothetical mean.
- Enter σ. (Remember, σ is known.)
- Enter $\overline{x}$.
- Enter n, the sample size.
- Select the type of alternative hypothesis.
- Select `Calculate` and press `ENTER`.

Hypothesis Testing on the TI-83

- A window appears with the following information.
- The title `Z-Test`.
- The alternative hypothesis.
- The value of the test statistic Z.
- The p-value of the test.
- The sample mean.
- The sample size.

Example

- Re-do Example 10.1 on the TI-83 using `Stats`.
- The TI-83 reports that
- $z = -2.575$.
- p-value $= 0.005012$.

Hypothesis Testing on the TI-83

- Suppose we had selected `Data` instead of `Stats`.
- Then somewhat different information is requested.
- Enter the hypothetical mean.
- Enter σ. (Why?)
- Identify the list that contains the data.
- Skip `Freq` (it should be `1`).
- Select the alternative hypothesis.
- Select `Calculate` and press `ENTER`.

Example

- Re-do Example 10.1 on the TI-83 using Data.
- Enter the data in the chart on page 616 into list $\mathtt{L}_1$.
- The TI-83 reports that
- $z = -2.575$.
- p-value $= 0.005012$.
- $\overline{x} = 12.528$.
- $s = 4.740$ ($\sigma = 4.8$).

Summary

- The seven steps are the same as in Chapter 9.
- If σ is known, then the test statistic is Z provided
  - The population is normal, or
  - The sample size n is at least 30.
- The TI-83 function Z-Test will perform steps 4 and 5 of this hypothesis test.